Response of a Liquid Column to Counterdirectional Excitation

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A finite cylindrical liquid column consisting of incompressible and frictionless liquid subjected to a counterdirectional axial harmonic excitation is modeled. The liquid system is assumed to be in a zero-gravity environment and held together by surface tension, which acts as the restoring force. The response of the system has been determined for a free surface elevation and velocity distribution. In addition, transient behavior was treated and damping introduced in the resonance terms. It was found that the first resonance response is sharply tuned and could easily be missed by a sweeping experiment. The investigation was performed for the design of the German D-2 Spacelab-mission "LICORE."

Nomenclature

a	= radius of liquid column
h	= length of liquid column
I_0, I_1	= modified Bessel functions
p	= liquid pressure
r, φ, z	= cylindrical polar coordinates
t	= time
u, w	= radial and axial velocity, respectively
v	= liquid velocity vector
$\bar{z}_0, \bar{z}_2$	= excitation amplitudes in axial direction
α_{2n}	$\equiv -\bar{\zeta}_{2n}\omega_{2n} + i\omega_{2n}\sqrt{1-\bar{\zeta}_{2n}^2}$
ρ	= liquid density
σ	= surface tension
Φ	= velocity potential
Φ_0	= velocity potential in time region $t<0$
Φ_1	= velocity potential in time region $t>0$
Φ_2	= final velocity potential for $t\to\infty$ (after transient has been damped out)
$\tilde{\Phi}$	= transient velocity potential
ζ	= liquid free surface displacement
ζ_0	= free liquid surface displacement in time region $t<0$
ζ_1	= free liquid surface displacement in time region $t>0$
ζ_2	= final free liquid surface displacement for $t\to\infty$ (after transient has been damped out)
$\tilde{\zeta}$	= transient free surface displacement
$\bar{\zeta}$	= damping factor of liquid
$\Omega, \Omega_0, \Omega_2$	= forcing frequencies
ω_{2n}	= natural frequencies

I. Introduction

THE vibration of cylindrical liquid bridges in microgravity or zero gravity has for recent years attracted the attention of experimenters trying to perform processes and experiments in an orbiting space laboratory.[1-3] The cylindrical column for such experiments is the most favored geometry. The oscillation of the cylinder's free liquid surface, which in the absence of gravity is held together by surface tension, is detrimental to the manufacturing process. Knowledge of the magnitude of the oscillatory frequencies of such a liquid column is of importance since it provides the range of frequencies to be avoided during the experiment or the manufacturing process. Such disturbances may appear as the result of the so-called g jitter of the space laboratory, the motion of its crew, the operation of onboard machines and instruments, or similar sources. For this reason, an experiment will be performed, under the title "Liquid Column Resonances" (LICORE), on the next German D-2 mission where the natural frequencies and response of a cylindrical liquid column will be studied experimentally.

In the course of these experiments, the liquid column shall be excited axially by a harmonic motion. To support these experiments, a theoretical model of the vibrational behavior of the liquid column is required before the flight in order to maximize the results of the orbital experiment. Linearized theory has been used previously[4-12] to determine the natural undamped and damped frequencies for frictionless, viscous, and even viscoelastic liquids. Even the nonlinear free oscillation case for a finite frictionless liquid column was recently computed.[13]

For the harmonic excitation of the complete liquid column,[14] the response has been determined to contain only the odd resonances. An experiment may, however, be performed using a mechanism with counterdirectional excitation, for which quite another set of resonances (i.e., even ones) appears. In this paper, the harmonic response corresponding to this excitation mode and the critical transient behavior appearing during frequency sweeping shall be investigated in detail.

II. Basic Equations

A cylindrical liquid bridge of diameter $2a$ and length h consists of incompressible and frictionless liquid (Fig. 1). It is excited in a counterdirectional fashion (i.e., the top, at $z=\frac{h}{2}$, undergoes displacement $-\bar{z}_0 e^{i\Omega t}$, while the bottom at $z=-\frac{h}{2}$ moves with $\bar{z}_0 e^{i\Omega t}$, where $\bar{z}_0$ is the excitation amplitude and Ω the forcing frequency). For this liquid column, the response of the liquid, which is held together by the liquid surface tension, has to be determined. Assuming irrotational flow, the velocity can be represented as the gradient of a velocity potential $\Phi(r,z,t)$, from which all mechanical values of interest may be obtained by differentiation or integration. Satisfying the continuity condition $\nabla \cdot \mathbf{v}=0$, the velocity potential ($\mathbf{v}=\nabla\Phi$) has to fulfill the Laplace equation:

$$\nabla^2\Phi=0 \tag{1}$$

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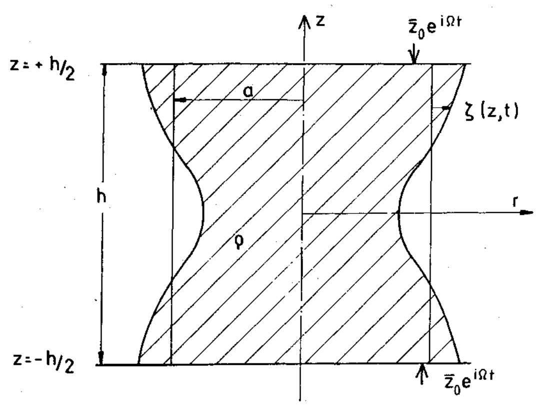

Fig. 1 Geometry and coordinates of liquid column.

where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ for axisymmetric motion. This equation has to be solved with the boundary conditions

$$\frac{\partial \Phi}{\partial z} = \pm\, i\Omega \bar{z}_0 e^{i\Omega t} \qquad \text{at } z = \mp \frac{h}{2} \tag{2}$$

at the bottom and the top, respectively. At the free liquid surface at $r = a$, the kinematic condition is

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \Phi}{\partial r} \qquad \text{at } r = a \tag{3}$$

where $\zeta(z,t)$ is the free liquid surface displacement above the equilibrium position $r = a$. The dynamic condition is given by

$$\frac{\partial \Phi}{\partial t} - \frac{\sigma}{\rho a^2}\left(\zeta + a^2 \frac{\partial^2 \zeta}{\partial z^2}\right) = 0 \qquad \text{at } r = a \tag{4}$$

which, when combined with the kinematic condition, yields

$$\frac{\partial^2 \Phi}{\partial t^2} - \frac{\sigma}{\rho a^2}\left(\frac{\partial \Phi}{\partial r} + a^2 \frac{\partial^3 \Phi}{\partial r \partial z^2}\right) = 0 \qquad \text{at } r = a \tag{5}$$

To obtain the velocity potential Φ, we have to solve Eqs. (1), (2), and (5) simultaneously.

III. Method of Solution

The free vibration problem for such a liquid column has been presented before.[12] It yields the eigenfunctions and the natural frequencies

$$\omega_{2n}^2 = \frac{\sigma}{\rho a^3}\frac{2n\pi a}{h}\left(\frac{4n^2\pi^2 a^2}{h^2} - 1\right)\frac{I_1\left(\frac{2n\pi a}{h}\right)}{I_0\left(\frac{2n\pi a}{h}\right)}, \quad n = 1,2,\ldots \tag{6}$$

where I_0 and I_1 are modified Bessel functions.

A. Response to Harmonic Excitation

For the solution of the counterdirectional excitation of the liquid column, we transform the velocity potential Φ with

$$\Phi(r,z,t) = e^{i\Omega t}\left[\Psi(r,z) - i\Omega \bar{z}_0 \frac{z^2}{h}\right] \tag{7}$$

which yields for the Laplace equation, Eq. (1), the Poisson equation

$$\nabla^2 \Psi = \frac{2i\Omega \bar{z}_0}{h} \tag{8}$$

with the homogeneous boundary conditions at the top and bottom of the column, i.e.,

$$\frac{\partial \Psi}{\partial z} = 0 \qquad \text{at } z = \pm \frac{h}{2} \tag{9}$$

The value $\bar{z}_0$ is the excitation amplitude in axial direction. The free surface condition, Eq. (5), yields

$$\Omega^2 \Psi + \frac{\sigma}{\rho a^3}\left(\frac{\partial \Psi}{\partial r} + a^2 \frac{\partial^3 \Psi}{\partial r \partial z^2}\right) = i\Omega^3 \bar{z}_0 \frac{z^2}{h} \qquad \text{at } r = a \tag{10}$$

A solution of Eq. (8) is given by

$$\Psi(r,z) = A_0(r) + \sum_{n=1}^{\infty} A_{2n}(r)\cos\left(\frac{2n\pi z}{h}\right) \tag{11}$$

which renders

$$A_0'' + \frac{1}{r}A_0' = \frac{2i\Omega \bar{z}_0}{h} \tag{12a}$$

where the double prime means the second derivative with respect to r and

$$A_{2n}'' + \frac{1}{r}A_{2n}' - \frac{4n^2\pi^2}{h^2}A_{2n} = 0 \tag{12b}$$

which, with the Fourier cosine expansion,

$$z^2 = \frac{h^2}{12} + \frac{h^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\left(\frac{2n\pi z}{h}\right)$$

have to satisfy the free surface condition, Eq. (10). That is,

$$\Omega^2 A_0 + \frac{\sigma}{\rho a^2}A_0' = \frac{i\Omega^3 \bar{z}_0 h}{12} \qquad \text{at } r = a \tag{13a}$$

and

$$\Omega^2 A_{2n} + \frac{\sigma}{\rho a^2}A_{2n}'\left(1 - \frac{4n^2\pi^2 a^2}{h^2}\right) = \frac{(-1)^n i\Omega^3 \bar{z}_0 h}{\pi^2 n^2} \quad \text{at } r = a \tag{13b}$$

The prime indicates differentiation with respect to r. The solutions are given by

$$A_0(r) = i\Omega \bar{z}_0 a\left[\frac{1}{2(h/a)}\left(\frac{r^2}{a^2} - 1\right) - \frac{\sigma}{\rho a^3 \Omega^2 (h/a)}\right] \tag{14a}$$

and

$$A_{2n}(r) = \frac{(-1)^{n-1} i\Omega^3 \bar{z}_0 h I_0\left(\frac{2n\pi r}{h}\right)}{\pi^2 n^2 I_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2 - \Omega^2)} \tag{14b}$$

The velocity potential $\Phi(r,z,t)$ is therefore given by the expression

$$\Phi(r,z,t) = i\Omega \bar{z}_0 e^{i\Omega t} a\left[\frac{1}{2(h/a)}\left(\frac{r^2}{a^2} - 1\right) - \frac{\sigma}{\rho a^3 \Omega^2}\frac{1}{(h/a)} - \frac{(z^2/a^2)}{(h/a)} + \frac{\Omega^2 h}{\pi^2 a}\sum_{n=1}^{\infty}\frac{(-1)^{n-1} I_0\left(\frac{2n\pi r}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{n^2 I_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2 - \Omega^2)}\right] \tag{15}$$

From this velocity potential, all mechanical values of interest may be obtained by differentiation or integration. The velocity distribution, for example, is given by

$$u(r,z,t)=\frac{\partial\Phi}{\partial r}=i\Omega\bar{z}_0e^{i\Omega t}\left[\frac{(r/a)}{(h/a)}+\frac{2\Omega^2}{\pi}\times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}I_1\left(\frac{2n\pi r}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{nI_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2-\Omega^2)}\right] \quad (16)$$

$$w(r,z,t)=\frac{\partial\Phi}{\partial z}=i\Omega\bar{z}_0e^{i\Omega t}\left[\frac{2(z/a)}{(h/a)}+\frac{2\Omega^2}{\pi}\times\sum_{n=1}^{\infty}\frac{(-1)^{n}I_0\left(\frac{2n\pi r}{h}\right)\sin\left(\frac{2n\pi z}{h}\right)}{nI_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2-\Omega^2)}\right] \quad (17)$$

The free surface elevation ζ is obtained from the kinematic condition, Eq. (3). It is

$$\zeta(z,t)=\bar{z}_0e^{i\Omega t}\left[\frac{1}{(h/a)}+\frac{2\Omega^2}{\pi}\times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{nI_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2-\Omega^2)}\right] \quad (18)$$

For very small forcing frequencies, $\Omega \ll 1$, the free surface elevation yields $\bar{z}_0e^{i\Omega t}/(h/a)$, indicating the change of the equilibrium location of the free surface.

B. Transient Response

For the determination of natural frequencies, a mechanical system is usually subjected to a forced sweeping experiment in which the forcing frequency is changed and the response of the system is measured. During such a change, the system shall change its motion from the original steady-state condition to another one. In doing this, it needs time to overcome the transient motion that is damped out in a short time period. The time duration for the transient to disappear is of great interest for the experimenter and necessary for the proper determination of the experimental results.

For this reason, we subject the liquid column, which is axially excited by $\bar{z}_0e^{i\Omega_0 t}$, at the time $t=0$ to another harmonic excitation $\bar{z}_2e^{i\Omega_2 t}$. To determine the transient motion of the liquid column, we have to solve the Laplace equation

$$\nabla^2\Phi_0=0 \qquad \text{for } t\leq 0 \quad (19a)$$

and

$$\nabla^2\Phi_1=0 \qquad \text{for } t\geq 0 \quad (19b)$$

with the boundary conditions

$$\frac{\partial\Phi_0}{\partial z}=\pm i\Omega_0\bar{z}_0e^{i\Omega_0 t} \qquad \text{at } z=\mp h/2 \text{ for } t\leq 0 \quad (20a)$$

$$\frac{\partial\Phi_1}{\partial z}=\pm i\Omega_2\bar{z}_2e^{i\Omega_2 t} \qquad \text{at } z=\mp h/2 \text{ for } t\geq 0 \quad (20b)$$

and the free surface condition

$$\frac{\partial^2\Phi_j}{\partial t^2}-\frac{\sigma}{\rho a^2}\left(\frac{\partial\Phi_j}{\partial r}+a^2\frac{\partial^3\Phi_j}{\partial r\partial z^2}\right)=0 \qquad \text{at } r=a\ (j=1,2) \quad (21)$$

The solution for the time period $t\leq 0$ is given by the previous results and yields a free surface displacement

$$\zeta_0(z,t)=\bar{z}_0e^{i\Omega_0 t}\left[\frac{1}{\left(\frac{h}{a}\right)}+\frac{2\Omega_0^2}{\pi}\times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{nI_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2-\Omega_0^2)}\right] \quad \text{for } t\leq 0 \quad (22a)$$

whereas for $t\rightarrow\infty$, we obtain the response of the final free surface displacement

$$\zeta_2(z,t)=\bar{z}_2e^{i\Omega_2 t}\left[\frac{1}{\left(\frac{h}{a}\right)}+\frac{2\Omega_2^2}{\pi}\times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{nI_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2-\Omega_2^2)}\right] \quad \text{for } t\rightarrow\infty \quad (22b)$$

The potential $\Phi_1(r,z,t)$ will compose of the final potential Φ_2 (r,z,t) and the transient potential $\bar{\Phi}$ (r,z,t). That is,

$$\Phi_1(r,z,t)=\Phi_2(r,z,t)+\bar{\Phi}^*(r,z,t)e^{i\omega_{2n}t} \quad (23)$$

The final free surface displacement $\zeta_2(z,t)$ would be reached for $t\rightarrow\infty$, provided the liquid exhibits a dissipative character. Since we treated the liquid as frictionless, its transient response will not decay, which is the reason for its time-dependency in Eq. (23). It is in a free oscillation. The solution of

$$\nabla^2\bar{\Phi}^*=0$$

with

$$\frac{\partial\bar{\Phi}^*}{\partial z}=0 \qquad \text{at } z=\pm h/2 \quad (24)$$

is given by

$$\bar{\Phi}^*(r,z)=\sum_{n=1}^{\infty}iA_{2n}I_0\left(\frac{2n\pi r}{h}\right)\cos\left(\frac{2n\pi z}{h}\right) \quad (25)$$

With the initial condition for the free surface displacement at $t=0$ given by

$$\zeta_0(z,0)=\zeta_2(z,0)+\bar{\zeta}(z,0) \quad (26)$$

we obtain, with the kinematic condition, the value of the integration constants A_{2n}:

$$A_{2n}=\frac{(-1)^{n-1}h\omega_{2n}}{\pi^2n^2I_0\left(\frac{2n\pi a}{h}\right)}\left(\frac{\bar{z}_0\Omega_0^2}{\omega_{2n}^2-\Omega_0^2}-\frac{\bar{z}_2\Omega_2^2}{\omega_{2n}^2-\Omega_2^2}\right)$$

which, together with $\zeta_2(z,t)$ and Eq. (25), yields the free surface displacement for the time range $t > 0$. It is

$$\zeta_1(z,t) = \frac{\bar{z}_0 e^{i\Omega_2 t}}{h/a} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1} I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{n I_0\left(\frac{2n\pi a}{h}\right)} \times \left[\frac{\bar{z}_2\Omega_2^2}{(\omega_{2n}^2 - \Omega_2^2)}\left(e^{i\Omega_2 t} - e^{i\omega_{2n} t}\right) + \frac{\bar{z}_0\Omega_0^2}{(\omega_{2n}^2 - \Omega_0^2)} e^{i\omega_{2n} t}\right] \quad \text{for } t \geq 0 \tag{27}$$

The velocity distribution, pressure, and liquid force may easily be obtained by differentiation or integration.

IV. Introduction of Damping

Since we are interested in maximum response amplitudes and since the liquid was treated here as frictionless, the response exhibits singularities in the resonance frequencies ω_{2n}. De facto, these amplitudes are, due to the viscosity of the liquid (not treated here), of finite magnitude. In order to account for this fact, we introduce in the resonance terms $(\omega_{2n}^2 - \Omega^2)$ a damping factor $\bar{\zeta}_{2n}$ that has to be determined by experiment. This term $(\omega_{2n}^2 - \Omega^2)$ has then to be replaced by $(\omega_{2n}^2 - \Omega^2 + 2i\bar{\zeta}_{2n}\Omega\omega_{2n})$. This means that the damped free surface elevation is given by

$$\zeta(z,t) = \bar{z}_0 e^{i\Omega_0 t}\left[\frac{1}{h/a} + \frac{2\Omega^2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1} I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{n I_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2 - \Omega^2 + 2i\bar{\zeta}_{2n}\Omega\omega_{2n})}\right] \tag{28}$$

the velocity distribution in radial direction by

$$u(r,z,t) = i\Omega\bar{z}_0 e^{i\Omega t}\left[\frac{r/a}{h/a} + \frac{2\Omega^2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1} I_0\left(\frac{2n\pi r}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{n I_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2 - \Omega^2 + 2i\bar{\zeta}_{2n}\Omega\omega_{2n})}\right] \tag{29}$$

and the velocity distribution in axial direction by

$$w(r,z,t) = i\Omega\bar{z}_0 e^{i\Omega t}\left[-\frac{2z/a}{h/a} + \frac{2\Omega^2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n} I_0\left(\frac{2n\pi r}{h}\right)\sin\left(\frac{2n\pi z}{h}\right)}{n I_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2 - \Omega^2 + 2i\bar{\zeta}_{2n}\Omega\omega_{2n})}\right] \tag{30}$$

In the case of a transient motion, the transient part will decay as

$$e^{-\bar{\zeta}_{2n}\omega_{2n}t + i\omega_{2n}\sqrt{1-\bar{\zeta}_{2n}^2}\, t} \equiv e^{\alpha_{2n}t}$$

for subcritical damping ($\bar{\zeta}_{2n} < 1$). The transient velocity potential is therefore given by

$$\bar{\Phi}(r,z,t) = \sum_{n=1}^{\infty} B_{2n} I_0\left(\frac{2n\pi r}{h}\right)\cos\left(\frac{2n\pi z}{h}\right) e^{\alpha_{2n}t}$$

Introducing this into the initial condition of the free surface displacement yields the magnitude of B_{2n}. The free surface displacement may therefore be expressed as

$$\zeta_0(z,t) = \bar{z}_0 e^{i\Omega_0 t}\left[\frac{1}{h/a} + \frac{2\Omega_0^2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1} I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{n I_0\left(\frac{2n\pi a}{h}\right)(\omega_{2n}^2 - \Omega_0^2 + 2i\bar{\zeta}_{2n}\Omega_0\omega_{2n})}\right] \tag{31}$$

for $t < 0$, and

$$\zeta_1(z,t) = \frac{\bar{z}_2 e^{i\Omega_2 t}}{h/a} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1} I_1\left(\frac{2n\pi a}{h}\right)\cos\left(\frac{2n\pi z}{h}\right)}{n I_0\left(\frac{2n\pi a}{h}\right)} \times \left[\frac{\bar{z}_2\Omega_2^2\left(e^{i\Omega_2 t} - e^{-\bar{\zeta}_{2n}\omega_{2n}t + i\omega_{2n}\sqrt{1-\bar{\zeta}_{2n}^2}\, t}\right)}{(\omega_{2n}^2 - \Omega_2^2 + 2i\bar{\zeta}_{2n}\Omega_2\omega_{2n})} + \frac{\bar{z}_0\Omega_0^2}{(\omega_{2n}^2 - \Omega_0^2 + 2i\bar{\zeta}_{2n}\Omega_0\omega_{2n})} e^{-\bar{\zeta}_{2n}\omega_{2n}t + i\omega_{2n}\sqrt{1-\bar{\zeta}_{2n}^2}\, t}\right] \tag{32}$$

where we obtain for increasing time ($t \to \infty$) the final response, Eq. (22b).

V. Numerical Evaluations and Conclusions

Some of the previously obtained analytical results have been evaluated numerically for some special parameters of the liquid system. The natural frequencies of Eq. (6), $\omega_{2n}/\sqrt{\sigma/\rho a^3}$, are presented as functions of the aspect ratio h/a of the liquid bridge. It may be noticed in Fig. 2 that with increasing height ratio h/a the natural frequencies decrease.

With increasing surface tension, they increase proportional to the square root of the surface tension parameter $\sigma/\rho a^3$. They are, additionally, larger for higher modes. One may also notice that, in the case of counterdirectional excitation, only the even natural frequencies ω_{2n} appear, whereas in the excitation mode of equal directional excitation of top and bottom of the liquid bridge only, the odd natural frequencies ω_{2n-1} appear. In the one-sided excitation mode, all natural frequencies ω_n are present. The response of the free surface elevation $|\zeta/\bar{z}_0 e^{i\Omega t}|$ is presented for the height ratio $h/a = 2$ for the surface tension parameter $\sigma/\rho a^3 = 1$ at the locations $z = 0$ and $h/4$ in Figs. 3a-3d. The results are given for $z = 0$ and various

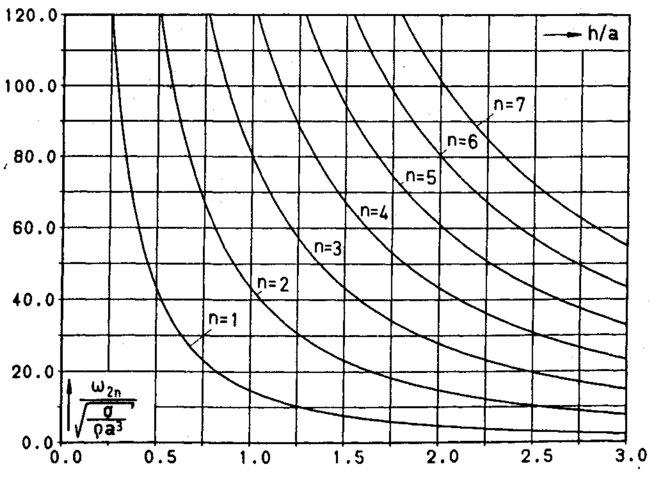

Fig. 2 Natural frequencies of circular cylindrical column.

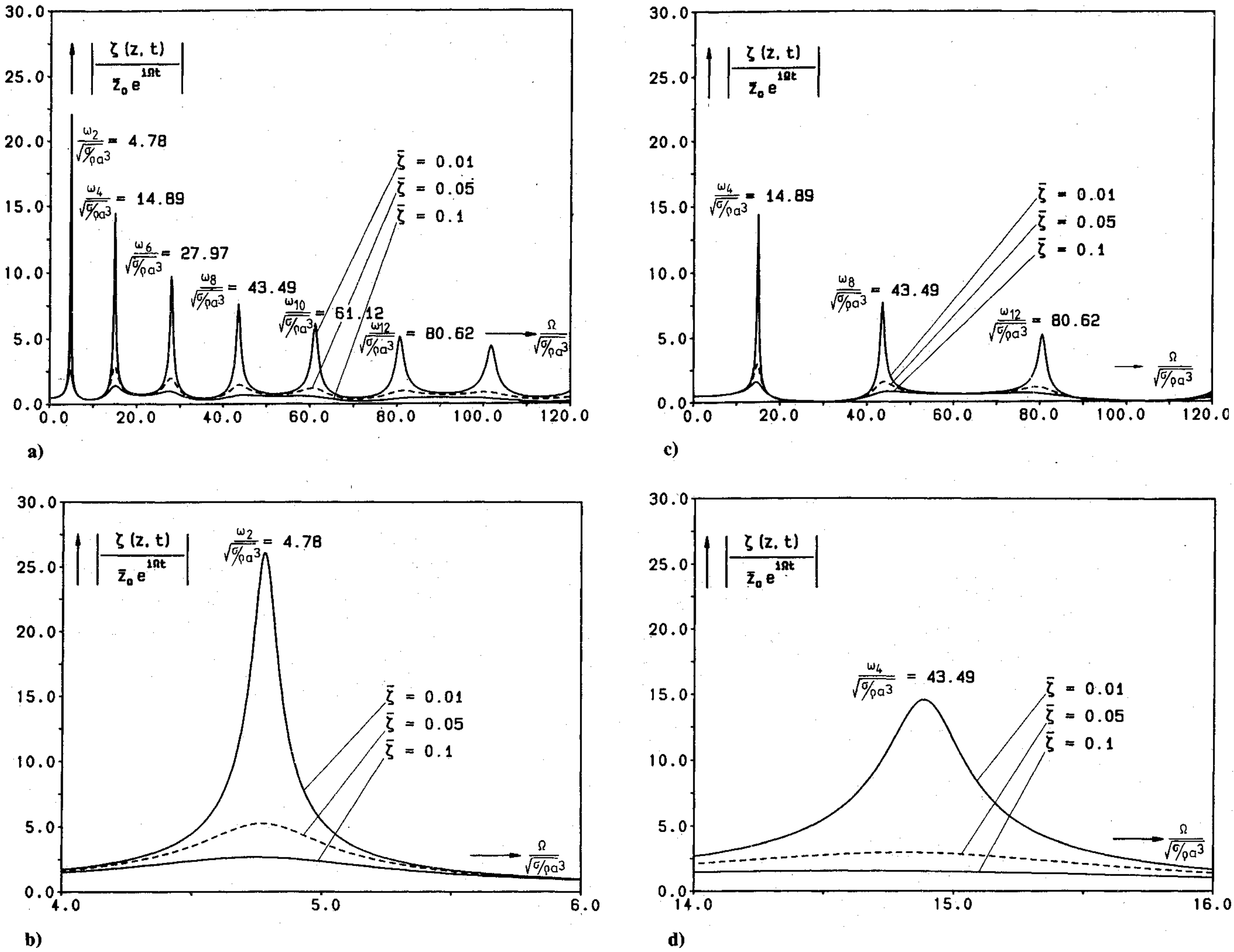

Fig. 3 Response of the free liquid surface elevation for a liquid column of length $h = 2a$; (a,b) at $z = 0$; and (c,d) at $z = h/4$.

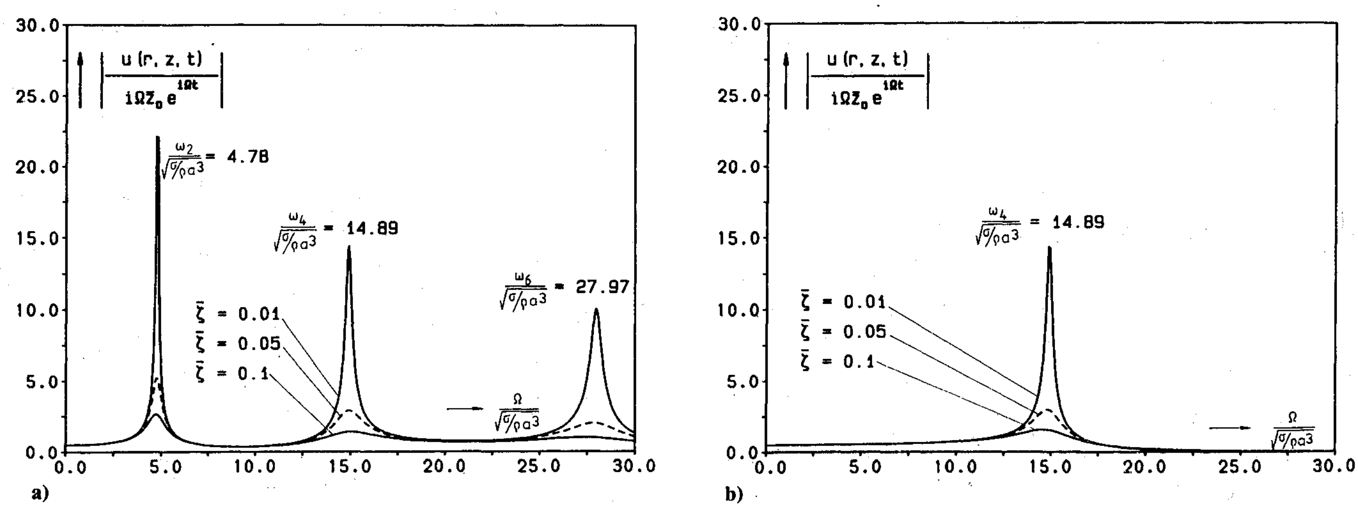

Fig. 4 Response of the radial velocity component for a liquid column of length $h = 2a$; (a) at $r = a$ and $z = 0$; and (b) at $z = h/a$.

damping factors $\bar{\zeta} = 0.01$, 0.05, and 0.1. We note first that the first resonance peak belonging to the second natural frequency ω_2 is quite large and sharp (Fig. 3a). The change of the damping factor from $\bar{\zeta} = 0.01$ to 0.05 reduces this peak from about $27\bar{z}_0$ to $5\bar{z}_0$. With increasing mode number, the resonance amplitudes decrease and show a wider response branch. For a more detailed presentation of the first resonance peak at ω_2, we show the response in Fig. 3b. Even a large damping of $\bar{\zeta} = 0.1$ still exhibits amplitudes of $\zeta \approx 2\bar{z}_0$. For $z = h/4$, the response is presented in Figs. 3c and 3d. We note here that the resonance frequencies ω_2, ω_6, etc., do not exhibit a response peak. This is due to the fact that $\cos(n\pi/2)$ vanishes for $n = 1$, 3, 5, etc. The motion of the liquid bridge is symmetric at $z = 0$. The response of the radial velocity $|u(r,z,t)/i\Omega\bar{z}_0 e^{i\Omega t}|$ is pre-

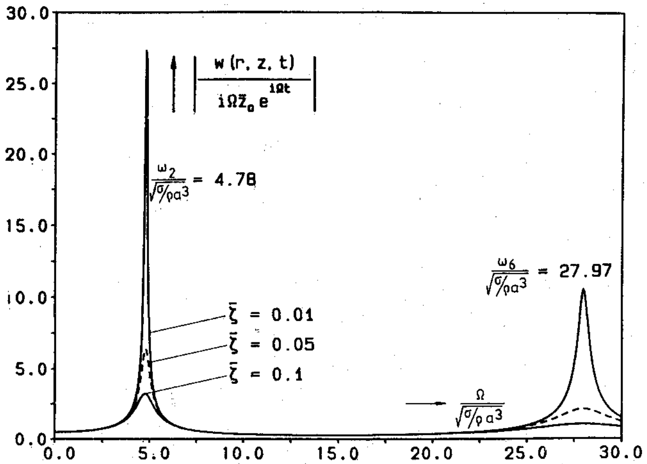

Fig. 5 Response of the axial velocity component for liquid column of length $h = 2a$ at the location $r = a$ and $z = h/4$.

sented in Figs. 4a and 4b for $\sigma/\rho a^3 = 1$ and at the locations $r/a = 1$, $z/h = 0$, and 1/4. Again, the velocity response in the radial direction shows a large peak at the first resonance (ω_2) which is, as in the previous cases, sharply tuned. At $r < a$, the amplitudes are smaller according to the radial dependency of the form of I_1 $(2n\pi r/h)$. At $z = h/4$, the resonance peaks at ω_2, ω_6, etc., are missing. Finally, we represent the axial velocity $|w(r,z,t)/i\Omega\bar{z}_0 e^{i\Omega t}|$ at the locations $z = h/4$ and $r/a = 1$ for $\sigma/\rho a^3 = 1$. For $z = 0$ and $z = h/2$, the axial velocity is zero and (-1), respectively. In Fig. 5, the axial velocity response is presented for $h/a = 2$ and $z = 1/4h$. Only the peaks at ω_2, ω_6, etc., appear due to the fact that sin $(n\pi/2)$ vanishes for even n values (i.e., $n = 4, 8, \ldots$). For more detail refer to the report in Ref. 15.

In conclusion, we may state the following:

1) The first resonance response ω_2 is sharply tuned and may in a sweeping experiment be missed by too fast a sweep procedure.

2) Only even natural frequencies appear in the counterdirectional excited liquid column. This is in contrast to the case where the bottom and top are forced in phase with the same amplitude. Then only odd natural frequencies are observed.

3) The damping of the liquid column decreases the resonance amplitudes and flattens the response behavior.

4) With increasing mode number, the resonance amplitudes decrease and the width of the response becomes wider.

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